Test Paper : **III**

Test Subject : **MATHEMATICAL SCIENCES**

Test Subject Code : **A-15-03**

Test Booklet Serial No. : ____________________

OMR Sheet No. : ____________________

Hall Ticket No. | | | | | | | | | |

(Figures as per admission card)

Name & Signature of Invigilator

Name : ______________________________ Signature : ______________________________

Paper : III

Subject : MATHEMATICAL SCIENCES

Time : 2 Hours 30 Minutes Maximum Marks : 150

Number of Pages in this Booklet : **16** Number of Questions in this Booklet : **75**

Instructions for the Candidates

1. Write your Hall Ticket Number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
   (iii) After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Sheet given to you**. If you mark at any place other than in the circle in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test question booklet and OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

అభ్యర్థులకు సూచనలు

1. ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్ టికెట్ నంబరు రాయండి.
2. ఈ ప్రశ్న పత్రము డెబ్బైఐదు బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
3. పరీక్ష ప్రారంభమున ఈ ప్రశ్నాపత్రము మీకు ఇవ్వబడుతుంది. మొదటి ఐదు నిమిషములలో **ఈ ప్రశ్నాపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా సరిచూసుకోండి.**
   (i) ఈ ప్రశ్న పత్రమును చూడడానికి కవర్‌పేజి అంచున ఉన్న కాగితపు సీలును చించండి. స్టిక్కర్ సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నాపత్రమును మీరు అంగీకరించవద్దు.
   (ii) **కవరు పేజి పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్నపత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏవైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇచ్చివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.**
   (iii) పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నాపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నాపత్రము పై నిర్దిష్టస్థలంలో రాయవలెను.
4. ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయ ప్రతిస్పందనలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతిప్రశ్నకు సరైన ప్రతిస్పందనను ఎన్నుకొని కింద తెలిపిన విధంగా OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన ప్రతిస్పందనను సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్‌తో కింద తెలిపిన విధంగా పూరించాలి.
   ఉదాహరణ : (A) (B) ● (D)
   (C) సరైన ప్రతిస్పందన అయితే
5. ప్రశ్నలకు ప్రతిస్పందనలను ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమాధాన పత్రంపై వేరొక చోట గుర్తిస్తే మీ ప్రతిస్పందన మూల్యాంకనం చేయబడదు.
6. ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
7. చిత్తుపనిని ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీస్థలములో చేయాలి.
8. OMR పత్రము పై నిర్ణీత స్థలంలో సూచించవలసిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అనర్హతకు మీరే బాధ్యులవుతారు.
9. పరీక్ష పూర్తయిన తర్వాత మీ ప్రశ్నపత్రాన్ని మరియు OMR పత్రాన్ని తప్పనిసరిగా పరీక్షపర్యవేక్షకుడికి ఇవ్వాలి. వాటిని పరీక్ష గది బయటకు తీసుకువెళ్లకూడదు.
10. **నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.**
11. **లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్‌లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్షగదిలో ఉపయోగించడం నిషేధం.**
12. **తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.**

MATHEMATICAL SCIENCES

Paper – III

1. The series $1+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$ converges if

(A) P < 1
(B) p > 1
(C) $p \leq 1$
(D) p = 0

2. If $\{f_n\}$ be a sequence of integrable functions defined on [a, b], then

(A) If $f_n \to 0$ a.e and each f_n is a bounded function, then $\int_a^b f_n(x)dx \to 0$

(B) If $f_n \to 0$ a.e and f_n are uniformly bounded functions, then $\int_a^b f_n(x)dx \to 0$

(C) If $\int_a^b f_n(x)dx \to 0$, then $f_n \to 0$ a.e

(D) If $f_n \to 0$ a.e, then $\int_a^b f_n(x)dx \to 0$

3. Suppose a and c are real numbers, c>0, and f is defined on [–1, 1] by

$f_n(x) = x^a \sin(x^{-c})$, if $x \neq 0$

= 0, if x = 0

Then which one of the following statements is true?

(A) f' is bounded $\Leftrightarrow a \leq 1 + c$
(B) f' is continuous $\Leftrightarrow a < 1 + c$
(C) $f'(0)$ exists $\Leftrightarrow a < 1$
(D) f is continuous $\Leftrightarrow a > 0$

4. If $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$, then

(A) f has discontinuities of the second kind at every point x

(B) f has discontinuities of the first kind at every point x

(C) $f(x_+)$ exists

(D) $f(x_-)$ exists

5. Compact subsets of metric spaces are

(A) convex

(B) open

(C) closed

(D) connected

6. Let E^0 denote the set of all interior points of a set E. Then which one of the statements is NOT true?

(A) E^0 is always open

(B) E is open if and only if $E^0 = E$

(C) E is closed

(D) If $G \subset E$ and G is open, then $G \subset E^0$

7. The Cayley Hamilton theorem states that

(A) The characteristic roots of a non-singular matrix are distinct

(B) The characteristic equation of a matrix admits a non-zero solution

(C) Every matrix satisfies its own characteristic equation

(D) The eigen values of any matrix are linearly independent

8. If A is an upper triangular matrix, then which one of the following statements is NOT true?

(A) A' is a lower triangular matrix

(B) |A| is the product of its diagonal elements

(C) Rank of A is equal to the order of A

(D) Eigen values of A are the diagonal elements of A

9. If α and β are two eigen values of a matrix A, then the corresponding eigen vectors of A are

(A) linearly dependent

(B) orthogonal

(C) orthonormal

(D) linearly independent

10. Which of the following map is NOT a linear transform?

(A) $T:V_3(R) \to V_2(R)$ defined by $T(x,y,z) = (x - y, x - z)$

(B) $T:V_3(R) \to V_1(R)$ defined by $T(x,y,z) = x^2 + y^2 + z^2$

(C) $D: V \to V$ defined by $D(f(x)) = df(x)/dx$, where V is a vector space of polynomials f(x) in x over R

(D) $I: V \to V$ defined by $I(f(x)) = \int_0^x f(x)\,dx$, where V is a vector space of polynomials f(x) in x over R.

11. If w is a subspace of $V_4(R)$ generated by the vectors (1,–2, 5, –3), (2, 3, 1, –4), (3, 8, –3, –5), then dim(w) =

(A) 1 (B) 2

(C) 3 (D) 4

12. A linear transform which reduces the quadratic form $x^2 + 4y^2 + 4z^2 + 4xy + 4xz + 16yz$ to the canonical form is

(A) $x = u + 2v,\ y = v - 2w,\ z = v + \frac{1}{2}w$

(B) $x = u - 2v,\ y = v + \frac{1}{2}w,\ z = v - \frac{1}{4}w$

(C) $x = u - 4v,\ y = v - \frac{1}{2}w,\ z = v + \frac{1}{2}w$

(D) $x = u + 4v,\ y = v + \frac{1}{2}w,\ z = v - \frac{1}{2}w$

13. If $u(x,y) = 2x - x^2 + ky^2$ is to be harmonic function, then k should be equal to

(A) 0

(B) 1

(C) 2

(D) 3

14. The bilinear transformation which maps 1, i, –1 to 2, i, –2 respectively is

(A) $w = \frac{2i - 6z}{iz - 3}$

(B) $w = \frac{zi - 3}{2i - 6}$

(C) $w = \frac{2i + 6z}{iz + 3}$

(D) $w = \frac{2i + 6z}{iz - 2}$

15. The value of $I = \oint_c \frac{z}{z^2 - 3z + 2} dz$ over $C: |z - 2| = \frac{1}{2}$ is

(A) $2\pi i$

(B) $4\pi i$

(C) $6\pi i$

(D) $8\pi i$

16. The Taylor series expansion of $f(z) = \frac{z-1}{z+1}$ about the point z =1 is

(A) $\frac{1}{2}(z-1) - (\frac{1}{2})^2(z-1)^2 + (\frac{1}{2})^3(z-1)^3 - \ldots$

(B) $\frac{1}{2}(z-1) + (\frac{1}{2})^2(z-1)^2 + (\frac{1}{2})^3(z-1)^3 + \ldots$

(C) $(z-1) - \frac{1}{2}(z-1)^2 + (\frac{1}{2})^2(z-1)^3 - \ldots$

(D) $(z-1) + \frac{1}{2}(z-1)^2 + (\frac{1}{2})^2(z-1)^3 + \ldots$

17. The residue of $f(z) = \frac{\sin z}{z \cos z}$ at the pole $z = \pi/2$ is

(A) 1

(B) $-2/\pi$

(C) $\pi/2$

(D) 0

18. If f(z) has a pole of order 3 at z = a, then Res {f(z)} =

(A) $\frac{1}{6}\frac{d^2}{dz^2}\left\{(z-a)^3 f(z)\right\}\Big|_{z=a}$

(B) $\frac{1}{2}\frac{d^2}{dz^2}\left\{(z-a)^3 f(z)\right\}\Big|_{z=a}$

(C) $\frac{1}{6}\frac{d^2}{dz^2}\left\{(z-a)^2 f(z)\right\}\Big|_{z=a}$

(D) $-\frac{1}{2}\frac{d^2}{dz^2}\left\{(z-a)^2 f(z)\right\}\Big|_{z=a}$

19. If $w = \log z$, then w is not analytic

(A) On real axis

(B) on negative real axis

(C) at $z=0$ only

(D) for complex z

20. If H and K are two subgroups of a group G, then HK is a subgroup of G if and only if

(A) $O(HK) = O(KH)$

(B) $HK = KH$

(C) $O(H) = O(K)$

(D) either $O(H)$ or $O(K)$ is a prime number

21. If H is a subgroup of G and $N(H) = \{ a \varepsilon G \mid a H a^{-1} = H\}$ then $N(H)$ is

(A) a normal subgroup of G

(B) a subgroup of G

(C) an abelian group

(D) a cyclic group

22. Let $O(G) = pq$ with p, q primes, $p > q$ where G is a group. Then which one of the following statements is NOT true?

(A) If $q \mid p-1$, then there exists a non-abelian group of order pq

(B) If $q \mid p-1$, then G is cyclic

(C) G has a subgroup of order p and a subgroup of order q

(D) Any two non-abelian groups of order pq are isomorphic

23. Let G be a group of order $11^2.13^2$. Then the number of 13 – Sylow subgroups in G are

(A) 1 (B) 2

(C) 3 (D) 4

24. If p is a prime number, then the splitting field over F, the field of rational numbers, of the polynomials $x^p - 1$ is of degree

(A) 2p+1

(B) p +1

(C) p – 1

(D) 2p – 1

25. Let I_1 be the ideal generated by $x^4 + 3x^2 + 2$ and I_2 be the ideal generated by $x^3 + 1$ in Q[x]. If $F_1 = Q[x]/I_1$ and $F_2 = Q[x]/I_2$, then

(A) F_1 is not a field while F_2 is a field

(B) neither F_1 nor F_2 is a field

(C) F_1 and F_2 are fields

(D) F_1 is a field but F_2 is not a field

26. The particular integral of the equation $yy' = y^2y' + (y')^2; y(0) = -\frac{1}{2}$ and $y'(0) = 1$ is

(A) $2y - 1 = 3y^2e^{x/2}$

(B) $2y - 1 = ye^{5x/2}$

(C) $2y - 3 = 8ye^{3x/2}$

(D) $2y + 3 = 6y^2e^{x/2}$

27. The singular solution of the equation $xp^2 - 2yp + 4x = 0$ is

(A) x (B) 2x

(C) 3x (D) 4x

28. Consider the boundary value problem: $y'' + \lambda y = 0, y(-\pi) = y(\pi), y'(-\pi) = y'(\pi)$ then to each eigen value λ there corresponds

(A) a unique eigen function

(B) two eigen functions

(C) two linearly independent eigen functions

(D) two mutually dependent eigen functions

29. Let $y_1(x)$ and $y_2(x)$ be a set of fundamental solutions of the equation

$y'' + p(x)y' + q(x)y = 0$ $a \le x \le b$, where p(x) and q(x) are continuous in [a, b] and $x_0 \varepsilon (a,b)$. Then which of the following statements is NOT true?

(A) $y_1(x)$ and $y_2(x)$ both can not have a local minimum at x_0

(B) $y_1(x)$ and $y_2(x)$ both can not have a local maximum at x_0

(C) $y_1(x)$ can have a local maximum at x_0 and $y_2(x)$ can have a local minimum at x_0

(D) $y_1(x)$ and $y_2(x)$ both can vanish simultaneously at x_0

30. A complete solution of the partial differential equation $\sqrt{p} + \sqrt{q} = 2x$ is

(A) $z = \frac{1}{4}(zx+a)^2 + a^2y + b$

(B) $z = \frac{1}{6}(zx+a)^3 + a^2y + b$

(C) $z = \frac{1}{6}(zx+a)^2 + ay + b$

(D) $z = \frac{1}{4}(zx+a) + ay + b$

31. A particular integral of the equation $(D^2 - D'^2)z = \cos(x + y)$ is

(A) $x \cos(x+y)$

(B) $\frac{1}{2} x \cos(x+y)$

(C) $x^2 \sin(x+y)$

(D) $\frac{1}{2} x \sin(x+y)$

32. A solution of the equation

$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, $u(0, y) = 8e^{-3y}$ is

(A) $u = 8\, e^{-12x-3y}$

(B) $u = 6\, e^{-4x-3y}$

(C) $u = 4\, e^{-4x-y}$

(D) $u = 2\, e^{-8x-3y}$

33. A real root of the equation $x \log_{10} x = 1.2$ correct to 3 decimal places obtained by Newton's iteration method , is

(A) 2.740 (B) 2.752

(C) 2.758 (D) 2.762

34. Given that $y' = -x^2 y$, $y(0) = 2$, then the value of y (0.2) obtained by modified Eulers method with h = 0.1 is

(A) 1.912 (B) 1.918

(C) 1.923 (D) 1.932

35. The approximate value of y(0.2) obtained upto 3 decimal places by solving $y' = x+y$, $y(0) = 1$ with h = 0.2 using the fourth order Runge - Kutta method, is

(A) 1.216

(B) 1.232

(C) 1.243

(D) 1.263

36. The shortest curve joining two fixed points on a given surface and lying entirely on that surface is called a

(A) cycloid

(B) geodesic

(C) catenary

(D) helix

37. A triangle enclosing the greatest area for a given perimeter, is

(A) an isosceles triangle

(B) an equilateral triangle

(C) a right angled triangle

(D) a scalene triangle

38. The condition that $\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)-\frac{\partial f}{\partial y}=0$ for the integral $I=\int_{x_1}^{x_2} f(x, y, y')dx$ to have a stationary value, is

(A) only a necessary condition

(B) a necessary and sufficient condition

(C) only a sufficient condition

(D) neither necessary nor sufficient condition

39. A particle of mass m with polar coordinates r, θ is subjected to a central force described by the potential v(r). Then its Lagrangian is

(A) $\frac{1}{2}m\dot{r}^2+\dot{\theta}^2-v(r)$

(B) $\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-v(r)$

(C) $\frac{1}{2}m(\dot{r}^2+\dot{\theta}^2)-v(r)$

(D) $\frac{1}{2}m(\dot{r}^2+\theta^2)-v(r)$

40. A body of mass m is moving in two dimensions with G as centre of mass. If v is the velocity of G and k is the radius of gyration of the body about a line passing through G, perpendicular to the plane of motion and θ is the angle between a line fixed in the body with a line fixed in space, then the kinetic energy of that body is

(A) $\frac{1}{2}m(v^2+k^2\dot{\theta}^2)$

(B) $m(v^2+k^2\dot{\theta}^2)$

(C) $\frac{1}{2}m(2v^2+k^2\theta^2)$

(D) $\frac{1}{2}m(v^2+2k^2\theta^2)$

41. The number of degrees of freedom of a particle moving in space, is

(A) 4 (B) 2

(C) 1 (D) 6

42. Let L be the Lagrangian of a conservative system under no constraints and q_k be a generalised coordinate. If $\frac{\partial L}{\partial q_k}=0$, then its generalised momentum is

(A) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)$

(B) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_k}\right)$

(C) constant of motion

(D) undefined

43. The eigen values of the integral equation

$y(x) = \lambda\int_0^1 (2xt - 4x^2)\, y(t)\mathrm{d}t$, are

(A) 2, 3 (B) 2, – 3

(C) 3, 3 (D) – 3, 3

44. The Voltera's equation of the second kind obtained by reducing the integral equation

$\int_0^x (2 + x^2 - t^2)\, y(t)\mathrm{d}t = x^2$, is y(x) =

(A) $2x - 2\int_0^x xy(t)\mathrm{d}t$

(B) $x + 2\int_0^x xy(t)\mathrm{d}t$

(C) $2x - 3\int_0^x xy(t)\mathrm{d}t$

(D) $x - 2\int_0^x xy(t)\mathrm{d}t$

45. The extremals of $\int_{-1}^{1} \left\{(y'')^2 + y\right\} dx$ subject to y(±1) =0 and y′(±1) =0, is

(A) $\frac{1}{24}(x^2 - 1)$ (B) $-\frac{1}{24}(x - 1)$

(C) $-\frac{1}{24}(x^2 - 1)^2$ (D) $\frac{1}{24}(x - 1)^2$

46. The mean absolute deviation is minimum when it is measured from

(A) mean

(B) median

(C) mode

(D) standard deviation

47. For a negatively skewed distribution, the correct relation between mean, median and mode is

(A) mean=median= mode

(B) mean < median < mode

(C) median< mean < mode

(D) mode < mean < median

48. With the usual notation Bowley's coefficient of skewness

(A) $\frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

(B) $\frac{Q_3 - Q_1 - 2M}{Q_3 + Q_1}$

(C) $\frac{Q_3 - Q_1 + 2M}{Q_3 - Q_1}$

(D) $\frac{Q_3 - Q_1 + 2M}{Q_3 + Q_1}$

49. If a random variable U is uniformly distributed over the interval (0, 1), then the distribution of $Y = \tan(\pi(U - 0.2))$ is

(A) Normal

(B) Laplace

(C) Cauchy

(D) Exponential

50. Which of the following is NOT the distribution function of a random variable?

(A) $F(x)=\begin{cases} 0 & if\ x<0 \\ 1-e^{-x} & if\ x\geq 0 \end{cases}$

(B) $F(x)=\begin{cases} 0 & if\ x<0 \\ e^{-x} & if\ x\geq 0 \end{cases}$

(C) $F(x)=\begin{cases} 0 & if\ x<0 \\ \frac{1}{2} & if\ 0\leq x<1 \\ 1 & if\ x\geq 0 \end{cases}$

(D) $F(x)=\begin{cases} 0 & if\ x<1 \\ 1 & if\ x\geq 1 \end{cases}$

51. The domain of a probability measure is

(A) the sample space

(B) the interval [0,1]

(C) the sigma field of subsets of the sample space

(D) the real line

52. Let $\{x_n\}$ be a Markovian chain on S= {1, 2, 3} with the following transition probability matrix : $P=\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$, then which of the following properties hold good for $\{x_n\}$?

(A) $\{x_n\}$ is irreducible

(B) all states are aperiodic

(C) all states are persistent

(D) all the above three

53. Identify in the following, a stochastic process that is an example of discrete time, discrete state space stochastic process

(A) Yule process

(B) renewal process

(C) Wiener process

(D) Branching process

54. If the mean of a binomial distribution is 3, then its variance could be

(A) 7.5

(B) 5.5

(C) 3.5

(D) 1.5

55. Let $\{x_n\}$ be a sequence of *i.i.d* non-negative r.v's with common mean θ and N be a Poisson r.v with mean λ, which is independent of x_i's. Then $E(x_1 + x_2 + \ldots + x_n) =$

(A) λ/θ

(B) $\lambda\,\theta$

(C) λ

(D) θ

56. If X and Y are two *i.i.d* standard normal variates then E(X/Y)=

(A) 1

(B) 0

(C) does not exist

(D) ½

57. With the usual notations the standard error of sample variance is

(A) $\sigma^2 \sqrt{(2/n)}$

(B) σ^2

(C) $\sigma^2 \sqrt{(n/2)}$

(D) $\sigma^2/2n$

58. Let $(x_1, x_2, \dots, x_n)$ be a random sample from $U(\theta-1, \theta+1)$. Then the M.L.E of θ is

(A) $x_{(1)}+1$

(B) $x_{(n)}-1$

(C) $\frac{1}{2}\left(x_{(1)}+x_{(n)}\right)$

(D) $\frac{1}{3}\left(x_{(n)}-1\right)+\frac{2}{3}\left(x_{(n)}+1\right)$

59. Let $x_1, x_2, \dots, x_n$ be a random sample from $N(\theta, \theta^2)$ and $T=\left(\Sigma_{i=1}^{n} x_i, \Sigma_{i=1}^{n} x_i^2\right)$. Then T is

(A) complete and sufficient for θ

(B) complete but not sufficient for θ

(C) not complete but sufficient for θ

(D) neither complete nor sufficient for θ

60. An example of a distribution which does not possess monotone likelihood ratio property is

(A) Normal

(B) Cauchy

(C) exponential

(D) Gamma

61. Neyman - Pearson most powerful test with power $1-\beta$ and size α satisfies

(A) $\alpha < 1-\beta$

(B) $\beta < 1-\alpha$

(C) $1-\beta < \alpha$

(D) $\alpha < \beta$

62. If the sample size in Wald - Wolfowitz runs test is large, the number of runs in the pooled ordered sample is distributed with mean

(A) $\frac{2m}{m+n}+1$

(B) $\frac{2n}{m+n}+1$

(C) $\frac{2mn}{m+n}$

(D) $\frac{2mn}{m+n}+1$

63. If two regression lines are perpendicular to each other then the relation between the two regression coefficients is

(A) $\beta_{xy} = \beta_{yx}$

(B) $\beta_{xy}\ \beta_{yx} = 1$

(C) $-\beta_{xy} = \beta_{yx}$

(D) $\beta_{xy} > \beta_{yx}$

64. The non-parametric test used for testing equality of more than two means of the population is

(A) Kruskal Wallis

(B) Median test

(C) Run test

(D) Rank test

65. With the usual notations $1 - R_{1.23}^2$ equals to

(A) $(1-\gamma_{12})(1-\gamma_{13.2}^{2})$

(B) $(1-\gamma_{12}^{2})(1-\gamma_{13.2})$

(C) $(1-\gamma_{12})(1-\gamma_{13.2})$

(D) $(1-\gamma_{12}^{2})(1-\gamma_{13.2}^{2})$

66. Confounding is a technique used for

(A) reducing the block size

(B) increasing the block size

(C) reducing the number of factors

(D) reducing the number of blocks

67. A sample of size n is drawn from a dichotomous population. If the sample has proportion p of items of category I and proportion q of category II then the variance of proportion is

(A) $s_p^2 = pq/(n-1)$

(B) $s_p^2 = pq/n$

(C) $s_p^2 = npq/(n-1)$

(D) $s_p^2 = p^2q/(n-1)$

68. For the linear programming problem, maximize z = x + 2y

subject to : $x + y \geq 1$, $x + 2y \leq 10$, $y \leq 4$, $x \geq 0$, $y \geq 0$

(A) Optimum solution is unique

(B) optimum solution exists but not unique

(C) optimum solution is unbounded

(D) optimum solution does not exist

69. The hazard function corresponding to the pdf: $f(x\,;\,\theta) = \theta x^{\theta-1} e^{-x^{\theta}}$, $x \geq 0$, $\theta > 0$ is

(A) always increasing in x

(B) always decreasing in x

(C) decreasing in x if $\theta<1$

(D) increasing in x if $\theta<1$

70. For a BIBD with b blocks such that each treatment occurs in r blocks, the rank of the incidence matrix N satisfies

(A) rank(N) = b – (r –1)

(B) rank(N) > b – (r – 1)

(C) rank(N) $\leq$ b – (r – 1)

(D) rank(N) = b+ (r – 1)

71. In the context of two group classification problem, the maximum likelihood allocation rule for assigning x_0 in terms of usual notation is

(A) assign x_0 to π_I if $\frac{f_1(x_0)}{f_2(x_0)} > 1$

(B) assign x_0 to π_I if $\frac{f_1(x_0)}{f_2(x_0)} > \frac{p_2}{p_1}$

(C) assign x_0 to π_I if $\frac{f_1(x_0)}{f_2(x_0)} > \frac{c(1/2)p_2}{c(2/1)p_1}$

(D) assign x_0 to π_I if $\frac{f_1(x_0)}{f_2(x_0)} > \frac{c(1/2)p_1}{c(2/1)p_2}$

72. In a two factor fixed -effects model with interaction, where A has ‘a’ levels and B has ‘b’ levels, we assume that

(A) $\Sigma_{i=1}^{a}\alpha_i = 0$, $\Sigma_{j=1}^{b}\beta_i = 0$, $\Sigma_{j=1}^{b}\gamma_{ij} = 0$, $\Sigma_{i=1}^{a}\gamma_{ij} = 0$

(B) $\Sigma_{i=1}^{a}\alpha_i = 0$, $\Sigma_{j=1}^{b}\beta_i = 0$, $\Sigma_{i=1}^{a}\gamma_{ij} = 0$

(C) $\Sigma_{i=1}^{a}\alpha_i = 0$, $\Sigma_{j=1}^{b}\beta_i = 0$, $\Sigma_{j=1}^{b}\gamma_{ij} = 0$

(D) $\Sigma_{i=1}^{a}\gamma_{ij} = 0$, $\Sigma_{j=1}^{b}\gamma_{ij} = 0$

73. The bath-tup curve type of hazard function is associated with

(A) Weibull distribution

(B) exponential distribution

(C) two component mixture of Weibull distributions

(D) Pareto distribution

74. If X and Y are two *i.i.d* r.v’s with mean 0 and variance σ^2, then $P[|x + y| > 2] \leq$

(A) σ^2

(B) $\sigma^2/2$

(C) $\sigma^2/4$

(D) $2\sigma - 2$

75. For economic order quantity models

i) the demand rate is constant always

ii) the replenishment rate is either finite or infinite

iii) the items are non-deteriorating only

Then

(A) i) is correct

(B) i) is not correct but ii) and iii) are correct

(C) ii) is correct

(D) i), ii) and iii) are correct

Space for Rough Work