SUBJECT CODE	SUBJECT	PAPER
A-15-02	MATHEMATICAL SCIENCES	II

HALL TICKET NUMBER	QUESTION BOOKLET NUMBER

OMR SHEET NUMBER

DURATION	MAXIMUM MARKS	NUMBER OF PAGES	NUMBER OF QUESTIONS
1 HOUR 15 MINUTES	100	8	50

This is to certify that, the entries made in the above portion are correctly written and verified.

Candidates Signature _____ **Name and Signature of Invigilator** _____

Instructions for the Candidates

1. Write your Hall Ticket Number in the space provided on the top of this page.
2. This paper consists of fifty multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested **to open the booklet and compulsorily examine it as below** :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
   (iii) After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example: (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Answer Sheet given to you**. If you mark at any place other than in the circle in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. **The candidate must handover the OMR Answer Sheet to the invigilators at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. The candidate is allowed to take away the carbon copy of OMR Sheet and used Question paper booklet at the end of the examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

అభ్యర్థులకు సూచనలు

1. ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్ టికెట్ నంబరు రాయండి.
2. ఈ ప్రశ్న పత్రము యాభై బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
3. పరీక్ష ప్రారంభమున ఈ ప్రశ్నాపత్రము మీకు ఇవ్వబడుతుంది. మొదటి ఐదు నిమిషములలో **ఈ ప్రశ్నాపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా సరిచూసుకోండి.**
   (i) ఈ ప్రశ్న పత్రమును చూడడానికి కవర్‌పేజి అంచున ఉన్న కాగితపు సీలును చించండి. స్టిక్కర్ సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నాపత్రమును మీరు అంగీకరించవద్దు.
   (ii) **కవరు పేజి పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్నపత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏవైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇచ్చివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.**
   (iii) పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నాపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నాపత్రము పై నిర్దిష్టస్థలంలో రాయవలెను.
4. ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయ ప్రతిస్పందనలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతిప్రశ్నకు సరైన ప్రతిస్పందనను ఎన్నుకొని కింద తెలిపిన విధంగా OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన ప్రతిస్పందనను సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్‌తో కింద తెలిపిన విధంగా పూరించాలి.
   ఉదాహరణ : (A) (B) ● (D)
   (C) సరైన ప్రతిస్పందన అయితే
5. ప్రశ్నలకు ప్రతిస్పందనలను ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమాధాన పత్రంపై వేరొక చోట గుర్తిస్తే మీ ప్రతిస్పందన మూల్యాంకనం చేయబడదు.
6. ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
7. చిత్తుపనిని ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీస్థలములో చేయాలి.
8. OMR పత్రము పై నిర్ణీత స్థలంలో సూచించబడిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అనర్హతకు మీరే బాధ్యులవుతారు.
9. **పరీక్ష పూర్తయిన తర్వాత మీ OMR పత్రాన్ని తప్పనిసరిగా పరీక్ష పర్యవేక్షకుడికి ఇవ్వాలి.** వాటిని పరీక్ష గది బయటకు తీసుకువెళ్లకూడదు. పరీక్ష పూర్తయిన తరువాత అభ్యర్థులు ప్రశ్న పత్రాన్ని, OMR పత్రం యొక్క కార్బన్ కాపీని తీసుకువెళ్లవచ్చు.
10. **నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.**
11. **లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్‌లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్షగదిలో ఉపయోగించడం నిషేధం.**
12. **తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.**

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MATHEMATICAL SCIENCES
Paper – II

1. If S is a set with 5 elements, then the number of binary operations that can be defined on S which are not commutative is

(A) $(1023)\, 2^{15}$ (B) $(1024)\, 2^{15}$

(C) 2^{15} (D) $2^{25} - 1$

2. The inverse of (1 3 5 7 9) o (5 7 9 2 4) o (2 4 6 8) in the group (S_9, o) is

(A) (1 3 5)o(9 2 7 4 6 8)

(B) (1 3 5 9)o(2 7 4 6 8)

(C) (1 3 5 9 2)o(7 4 6 8)

(D) (1 3 5 9 2 7)o(4 6 8)

3. Let ω be the complex cube root of unity and $G = \{1, \omega, \omega^2\}$. Let ϕ be a homomorphism from the group $(\mathbb{Z}, +)$ into the group $(G, \cdot)$ defined by $\phi(n) = \omega^n$ for all $n \in \mathbb{Z}$. Then Ker ϕ =

(A) $\mathbb{Z}$ (B) $2\mathbb{Z}$

(C) $3\mathbb{Z}$ (D) $4\mathbb{Z}$

4. If X = {1, 2, 3, 4}, then which one of the following is NOT a topology on X ?

(A) $\tau_1 = \{\phi, X, \{1,2\}, \{2, 3\}, \{1, 2, 3\}\}$

(B) $\tau_2 = \{\phi, X, \{3\}, \{1,3\}\}$

(C) $\tau_3 = \{\phi, X, \{1\}, \{1, 2\}, \{1, 2, 3\}\}$

(D) $\tau_4 = \{\phi, X, \{1\}\}$

5. Which one of the following is NOT a dense subset of $\mathbb{R}$ in the standard topology on $\mathbb{R}$?

(A) $\mathbb{Q}$ (B) $\mathbb{R} - \mathbb{Q}$

(C) $\mathbb{Z}$ (D) $\mathbb{R} - \mathbb{Z}$

6. An example of a connected subset of $\mathbb{R}$ (in the standard topology) which is NOT compact is

(A) [0, 1] (B) (0, 1)

(C) $(0, 1) \cup (1, 2)$ (D) $[0, 1] \cup [1, 2]$

7. If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined by T(x, y, z) = (x + y, y + z, z – x) and if B is the standard basis for $\mathbb{R}^3$, then the matrix [T : B : B] =

(A) $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

8. In the vector space $(\mathbb{R}^3, +, \cdot)$ over $\mathbb{R}$, which one of the following is a linearly independent set ?

(A) {(1, 2, 3), (2, –1 , 4), (4, 3, 10)}

(B) { (2, 1, 7), (3, – 2, 4), (–4, 1, –1)}

(C) {(1, – 3, 2),(3, –1, 1), (1, 5, –3)}

(D) {(2, – 1, 4), (3, 1, 4), (1, –3, 4)}

9. If V is the vector space of all 4 × 4 matrices over $\mathbb{R}$, then the dimension of the vector space V over $\mathbb{R}$ is

(A) 2 (B) 4

(C) 8 (D) 16

10. Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by T(x, y, z) = (x – y, y – z) for all $(x, y, z) \in \mathbb{R}^3$, then nullity of T =

(A) 3 (B) 2

(C) 1 (D) 0

11. The number of solutions of the non-homogeneous system of equations given by

$x + 2y - z = 4$

$2x - 3y + z = 3$

$5x - 4y + z = 5$, is

(A) 0 (B) 1

(C) 2 (D) infinite

12. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^5}$

(A) converges

(B) diverges

(C) absolutely converges

(D) conditionally converges

13. Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x^2 \sin x, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}, \text{ then}$$

(A) f is discontinuous at x = 0

(B) f is continuous but not differentiable at x = 0

(C) $\lim_{x \to 0} f(x)$ does not exist

(D) f is differentiable at x = 0

14. $\int_0^{\frac{\pi}{2}} \ln \sin x \, dx =$

(A) $\frac{\pi}{2} \ln 2$ (B) $-\frac{\pi}{2} \ln 2$

(C) ln 2 (D) does not exist

15. The Lebesgue measure of the set $\mathbb{Z}$ of all integers

(A) is infinite (B) is equal to 1

(C) is equal to 0 (D) lies in (0, 1)

16. Which one of the following statements is FALSE ?

(A) The Lebesgue measure of an interval is its length

(B) The set of all Lebesgue measurable sets is a σ - algebra

(C) Every Borel set is Lebesgue measurable

(D) Every measurable function is Lebesgue integrable

17. If z = 2 + 3i, the modulus of the complex number $\frac{7z-1}{3z+1}$ is

(A) $\frac{1}{13}(28 + 3i)$ (B) $\frac{1}{130}(28 + 3i)$

(C) $\frac{1}{13}(280 + 30i)$ (D) $\frac{1}{130}(280 + 3i)$

18. $\int_0^{1+i} (x + iy)\,dz$ along y = x is

(A) i (B) 0

(C) 1 (D) 1 + i

19. A critical point of $w = z^2$ is

(A) 1 (B) – 1

(C) 0 (D) ∞

20. If C is the circle $x^2 + y^2 = 4$, then $\oint_C \frac{3z^2 + 7z + 1}{z - 3} dz =$

(A) 6i (B) 0

(C) –6i (D) i

21. Given the following tabular data :

x	**:**	–1	0	2	3
y = f(x)	**:**	– 8	3	1	2

The approximate value of f(1) correct to three decimal places is

(A) 3.447 (B) 3.538

(C) 3.666 (D) 3.764

22. The approximate value of the integral $\int_0^6 \frac{dx}{1+x^2}$ by trapezoidal rule with six intervals is

(A) 1.3662 (B) 1.4108

(C) 1.1048 (D) 1.6332

23. If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1y' + a_2y = 0$ (a_1, a_2 are constants) on an interval I containing a point x_0, then the Wronskian $W(\phi_1, \phi_2)$ of the solutions at x is

(A) $W(\phi_1,\phi_2)(x) = e^{a_1(x-x_0)}W(\phi_1,\phi_2)(x_0)$

(B) $W(\phi_1,\phi_2)(x) = e^{-a_1(x-x_0)}W(\phi_1,\phi_2)(x_0)$

(C) $W(\phi_1,\phi_2)(x) = e^{-a_1(x-x_0)}W(\phi_1,\phi_2)(0)$

(D) $W(\phi_1,\phi_2)(x) = e^{a_1(x-x_0)}W(\phi_1,\phi_2)(0)$

24. The Sturm Liouville operator L in the equation $L(y) + \lambda\rho y = 0$ is

(A) $p\frac{d^2}{dx^2} - q$ (B) $q\frac{d}{dx} - p$

(C) $\frac{1}{p}\frac{d^2p}{dx^2} - \frac{1}{q}\frac{dq}{dx}$ (D) $\frac{d}{dx}\left(p\frac{d}{dx}\right) - q$

(p and q are some functions of x)

25. An integrating factor for the equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ is

(A) $(x+1)e^{3x}$ (B) $-(x+1)$

(C) $-\left(\frac{1}{x^2+1}\right)$ (D) $\frac{1}{x+1}$

26. With the usual notation, the principle of least action in the Jacobi's form, is

(A) $\delta\int_{t_1}^{t_2}\sqrt{H-V}\,dt = 0$

(B) $\Delta\int_{\rho_1}^{\rho_2}\sqrt{H-V}\,d\rho = 0$

(C) $\delta\int_{\rho_1}^{\rho_2}\sqrt{H+V}\,d\rho = 0$

(D) $\Delta\int_{\rho_1}^{\rho_2}\sqrt{T-V}\,d\rho = 0$

27. The kinetic energy of a particle of mass 'm' moving in a plane, in polar coordinates r and θ is

(A) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$ (B) $\frac{1}{2}m(r^2 + r^2\dot{\theta}^2)$

(C) $\frac{1}{2}m(\dot{r}^2 + \frac{1}{r^2}\dot{\theta}^2)$ (D) $\frac{1}{2}m\left(\dot{r} + \frac{1}{r}\dot{\theta}^2\right)$

28. A solution of the integral equation $y(x) = \sin x + 2\int_0^x \cos(x-t)y(t)\,dt$ is

(A) xe^x (B) x^2e^x

(C) $e^x \sin x$ (D) $\frac{e^x}{x}$

29. The curve on which the functional $\int_0^1 \left[(y')^2 + 12xy\right]dx;\ y(0) = 0$ and $y(1) = 2$ can be extremized, is

(A) $y = x^3$

(B) $y = 2x^3 - x$

(C) $y = x^3 + x^2 + 1$

(D) $y = x^3 + x$

30. The particular integral of

$(D^2 - DD')z = \cos x \cos 2y$, with the usual notation, is

(A) $\frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$

(B) $\frac{1}{3}\cos(x+2y) + \frac{1}{2}\cos(x-2y)$

(C) $\frac{1}{2}\cos(2x+y) - \frac{1}{3}\sin(x-2y)$

(D) $\frac{1}{3}\cos(x+2y) + \frac{1}{6}\cos(x-2y)$

31. To understand the skewness of the variable which diagram is used ?

(A) Pictogram

(B) Whisker diagram

(C) Line

(D) Range chart

32. Let $\{X_n\}$ be a sequence of iid r.v.s with mean $\mu < \infty$. Then $\overline{X}_n \to \mu$ in probability. This statement relates to

(A) Chebychev's WLLN

(B) Khintchine's WLLN

(C) Kolmogorov's SLLN

(D) Central limit theorem

33. For a market research survey, a sample of 10 retail outlets are selected. The chances for selecting a hyper market, super market, departmental store and a restaurant are 0.10, 0.50, 0.20 and 0.20 respectively. What is the probability that at least three super markets are selected ?

(A) $1 - \frac{7}{2^7}$ (B) $\frac{2}{2^7}$

(C) $\frac{7}{2^7}$ (D) $\frac{1}{2^{10}}$

34. If A and B are independent non-null events then which one of the following is NOT correct ?

(A) $P(A \cap B) = P(A) \cdot P(B)$

(B) $P(A \cup B) = 1 - P(A^C)P(B^C)$

(C) $P(A^C \cap B) = P(A^C) \cdot P(B)$

(D) $P(A \cup B) = P(A) + P(B)$

35. If $p_{ii}^{(n)} = 1$ for all values of n, then the state i is called _____ state.

(A) Reflecting (B) Absorbing

(C) Communicating (D) Periodic

36. If T is the Transition Probability Matrix (TPM) of a Markov Chain, then the correct statement is

(A) T^2 is again a TPM

(B) $T^2 + T$ is again a TPM

(C) αT is again a TPM for $0 < \alpha < 1$

(D) log T is again a TPM

37. If X is a standard normal r.v. then the ratio of quartile deviation, mean deviation and standard deviation is

(A) 5 : 6 : 7 (B) 10 : 12 : 15

(C) 2 : 3 : 4 (D) 1 : 2 : 3

38. If X is a Poisson r.v. with mean $\frac{3}{2}$ then its variance is

(A) $\frac{2}{3}$ (B) 3

(C) $\frac{3}{2}$ (D) 2

39. Let $X_1, X_2, \ldots, X_n$ be a random sample form a normal population with mean μ and variance σ^2. If $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$, then the unbiased estimator for σ^2 is

(A) $\frac{1}{n-1}s^2$ (B) $\frac{1}{n}s^2$

(C) $\frac{n-1}{n}s^2$ (D) $\frac{n}{n-1}s^2$

40. A sample of 50 observations drawn from a normal population with mean μ and variance σ^2. The sample mean and standard deviation are 11,795 and 14,054 respectively. What is the 95% confidence interval for μ ?

(A) (11795, 15691) (B) (7855, 15291)
(C) (3896, 11795) (D) (12560, 13480)

41. Local control is the concept related to ___

(A) The reduction of experimental limits
(B) The reduction of treatments
(C) The reduction of heterogeneity in the experimental material
(D) The reduction of the cost of the experiment

42. The length of 5% confidence interval is _____ than the length of 1% confidence interval.

(A) Larger
(B) Less than or equal to
(C) Greater than or equal to
(D) Smaller

43. The ratio estimator is more precise than SRSWOR estimator for the populations total if

(A) $p > \frac{1}{2}\frac{C_x}{C_y}$ (B) $p > \frac{C_x}{C_y}$

(C) $p > \frac{1}{2}\frac{C_y}{C_x}$ (D) $p > \frac{C_y}{C_x}$

44. In sample surveys one can have the following error

(A) sampling error
(B) non-sampling error
(C) both sampling and non-sampling errors
(D) no error

45. The $X'AX$ and $X'BX$ are independent quadratic forms of normal vectors if and only if

(A) $AB = 0$ (B) $AB \neq 0$
(C) $A = B$ (D) $A = B^{-1}$

46. Let $f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0 & elsewhere \end{cases}$, then the marginal density of Y is

(A) $4y^3$ (B) y^3
(C) $4x^2$ (D) $4y$

47. A linear combination of treatments said to be contrast if and only if

(A) The sum of the treatment effects is zero
(B) All the coefficients of the treatments are unity
(C) The sum of coefficients of the treatments is zero
(D) The number of positive and negative coefficients is same

48. The total number of factorial effects in 2^n experiment are

(A) 2^n
(B) $1 - 2^n$
(C) $2^n - 1$
(D) $2^n + 1$

49. Which of the following statement is NOT true ?

(A) degeneracy in LPP may arise at the initial stage
(B) a degenerate solution can never be optimum
(C) degeneracy may be a temporary phenomenon
(D) LPP can be used in solving a game

50. In an M/M/1 queuing model under equilibrium the mean arrival rate is 3 and mean service rate is 4. What is the probability that the server is busy ?

(A) 0.25 (B) 0.75
(C) 0.5 (D) 0

Space for Rough Work